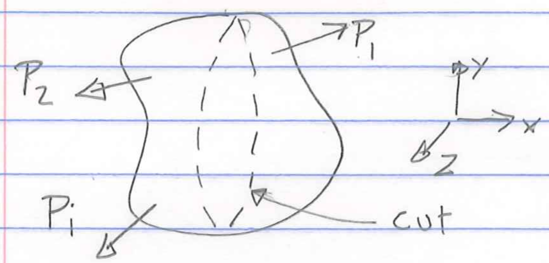
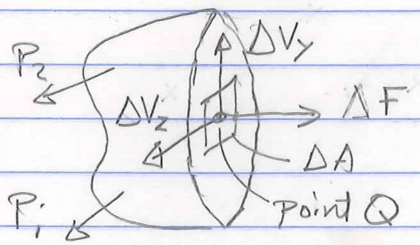
1. Stress definition

Consider a body loaded with forces



Cut the body along a plane parallel to Y-Z. Examine the part of the body to the left of the cut with a FBD.



x is $\perp$ to plane of cut; y, z are parallel. ΔF, ΔV_y and ΔV_z are the x, y, z components of the resultant force on the small area ΔA. (ΔF is a normal component, and ΔV_y and ΔV_z are shear components.)

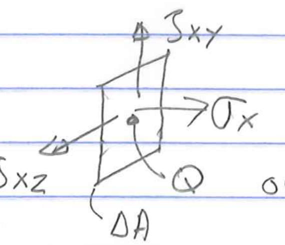
Shrink the area ΔA around point Q.

Define the $\lim_{\Delta A \to 0} \frac{\Delta F}{\Delta A}$ as the normal stress σ_x at point Q.
axis $\perp$ to ΔA

Define the $\lim_{\Delta A \to 0} \frac{\Delta V_y}{\Delta A}$ as the shear stress τ_{xy} at point Q.
axis $\perp$ to ΔA — component of shear stress

Define the $\lim_{\Delta A \to 0} \frac{\Delta V_z}{\Delta A}$ as the shear stress τ_{xz} at point Q.
axis $\perp$ to ΔA — component of shear stress

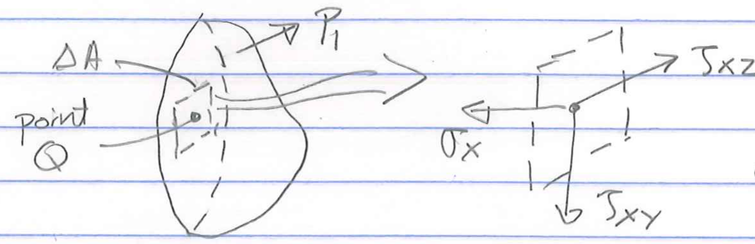
The above stresses can be depicted as:



These are stresses on the +X face passing through Q. (The outward normal to a +X face is in the +X direction.)

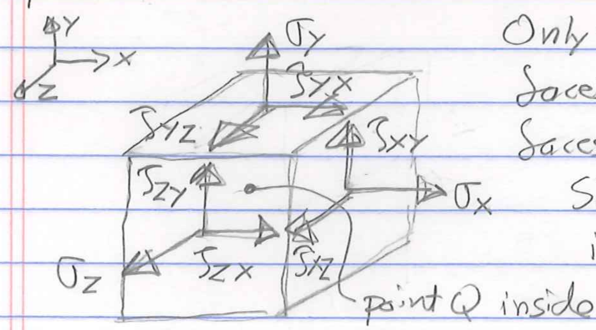
Stress is a vector and the quantities σ_x, τ_{xy}, τ_{xz} are the x, y, z components of the stress vector acting on ΔA. The arrows show the direction of the components, but the stresses are actually distributed over ΔA. As $\Delta A \to 0$, the distributions of stress approach being uniform, which are the stresses at point Q.

If we had considered the right-side free body, the same stresses would have been obtained at Q, but in opposite directions.



Stresses on the $-x$ face passing through Q. (The outward normal to a $-x$ face is in the $-x$ direction.)

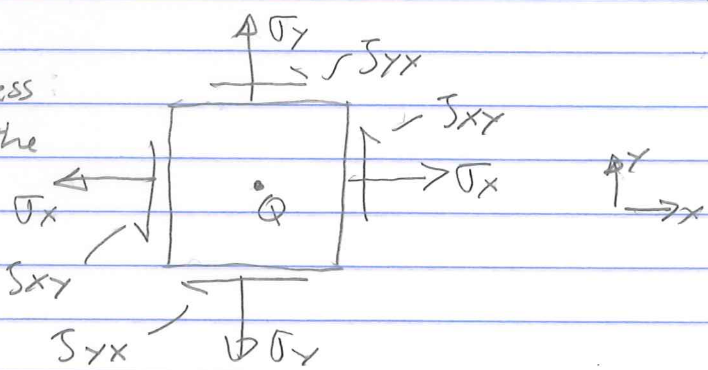
Repeating the above limiting process on a small cube around point Q reveals additional stress components:



Only the stress components on the $+x, +y, +z$ faces are shown; ones on the $-x, -y, -z$ faces (hidden) are not shown.

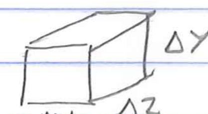
Sign convention: positive stresses are in the $+x, +y, +z$ directions on $+$ faces, and in the $-x, -y, -z$ directions on $-$ faces. For the τ stresses, the first subscript denotes the face and the second denotes the component.

For a 2-d state of stress (plane stress), say, for the x-y plane:
All stresses are shown according to the positive sign convention.



As indicated, the 3-d state of stress at a point can be expressed with respect to x, y, z by 9 components (3 normal, 6 shear). However, this description can be reduced to 3 normal and 3 shear as follows:

$\Sigma \bar{M} = 0$ about any axis for static equilibrium.



$$\Sigma M_y = \tau_{xy} \cdot \Delta y\, \Delta z \cdot \Delta x - \tau_{yx} \cdot \Delta x\, \Delta z \cdot \Delta y$$

$$= 0 \implies \tau_{xy} = \tau_{yx},$$ where it is assumed

that $\Delta x, \Delta y, \Delta z$ are small enough so that the stresses can be assumed to be uniform.

Similarly, $\Sigma M_x = 0 \Rightarrow \tau_{yz} = \tau_{zy}$

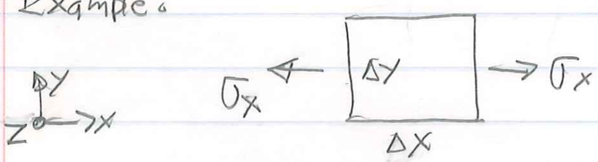
and $\Sigma M_y = 0 \Rightarrow \tau_{xz} = \tau_{zx}$

The x, y, z stress state is usually referred to as $\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{xz}, \tau_{yz}$. (3 normal, 3 shear)

For plane stress, say in the x-y plane, the x, y stress state is $\sigma_x, \sigma_y, \tau_{xy}$ (2 normal, 1 shear).

The x, y, z stress state is sufficient to determine the stress components at a point on any plane at any orientation.

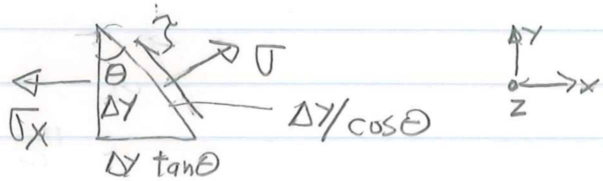
Example:



Only $\sigma_x \neq 0$.

$\Delta Z \perp$ to page

Want to find σ, τ



$$\Sigma F_x = 0: \ -\sigma_x \cdot \Delta Y \, \Delta Z + \sigma \cos\theta \frac{\Delta Y}{\cos\theta} \Delta Z - \tau \sin\theta \frac{\Delta Y}{\cos\theta} \Delta Z = 0$$

$$\Sigma F_y = 0: \ \sigma \sin\theta \frac{\Delta Y}{\cos\theta} \Delta Z + \tau \cos\theta \frac{\Delta Y}{\cos\theta} \Delta Z = 0$$

Solve: $\sigma = \sigma_x \cos^2\theta \ ; \ \tau = -\sigma_x \cos\theta \sin\theta$

All of the above concepts of stress apply equally well to dynamic problems.